Enhanced triplet Andreev reflection off a domain wall in a lateral geometry

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We find that the triplet Andreev reflection amplitude at the interface between a half-metal and an *s*-wave superconductor in the presence of a domain wall is significantly enhanced if the half-metal is coupled laterally to the superconductor. Whereas triplet Andreev reflection is absent at the Fermi energy in the case of serial coupling, it is nonzero in a lateral contact geometry. We also find that in the lateral case domain walls cause (Andreev) backscattering even in the adiabatic limit of long domain walls, contrary to adiabatic domain walls in ordinary magnetic systems. For a lateral contact, domain walls can thus be an effective source of the triplet proximity effect.

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I. INTRODUCTION

A normal metal inherits superconducting properties if it is in electrical contact to a superconductor. This "superconductor proximity effect" is mediated by Andreev reflection,[1] the process in which an electron incident from the normal metal is reflected as a hole at the normal-metal superconductor interface. As phase coherence between the electron and the Andreev reflected hole is preserved over long distances $\sim\hbar v_F/T$, where v_F is the Fermi velocity and T the temperature, superconducting correlations extend deep into the normal metal.

At the interface between a ferromagnet and a superconductor, majority electrons (electrons with their spin parallel to the magnetization direction $\mathbf{m}$) are Andreev reflected as minority holes and vice versa.[2] With the relative phase between majority electrons and minority holes now set by the exchange energy of the ferromagnet instead of the much smaller excitation energy of electron and hole, the proximity effect becomes effectively short range in a ferromagnet. The situation is even more extreme in a half-metal, a material in which only majority charge carriers exist. At a half-metal superconductor interface, Andreev reflection of majority electrons is strongly suppressed simply because of the absence of minority holes.

It was realized by Bergeret *et al.*[3] (see also Ref. 4) that the situation is entirely different if spin-rotation symmetry around the (mean) magnetization direction at the superconductor interface is broken: In that case, majority electrons may be reflected as majority holes. The (odd-frequency) "triplet proximity effect" that results from such "spin-flip" Andreev reflection can penetrate ferromagnets or half-metals the same distance as the standard proximity effect penetrates normal metals.[5] Various experiments have hinted at the existence of this effect,[6–9] the most striking of which is the observation of a Josephson current through a μm-long link of the half-metal CrO_2 by Keizer *et al.*[7]

There have been various proposals for the origin of the broken spin-rotation symmetry needed for the existence of the long-range triplet proximity effect. One possibility is an artificial structure in which there is a thin ferromagnetic or half-metallic spacer layer at the interface with a magnetization direction different from that of the bulk magnet.[10,11] For this scenario the ferromagnetic spacer layer should be thin enough that the standard proximity effect has a range larger than its thickness. A second possibility is a magnetically disordered or "spin-active" interface.[12,13] Finally, the triplet proximity effect can be caused by variations in the magnetization direction $\mathbf{m}$ associated with a domain wall, either perpendicular[14] or parallel to the superconductor interface.[15]

In this paper we focus on the triplet proximity effect in the presence of a domain wall in a half-metallic film. The case of a half-metal is not only most relevant for the experiment of Ref. 7, it also allows for an unambiguous identification of the triplet proximity effect:[12] in the absence of minority carriers, spin-conserving Andreev reflection of majority electrons into minority holes is ruled out, and the "spin-flip" Andreev reflection associated with the triplet proximity effect is the only possible Andreev reflection process in a half-metal.

Following Ref. 11 we employ a scattering approach which allows the treatment of exchange fields of arbitrary strength, in particular the half-metallic case. While a distinction of odd- and even-frequency contributions to the triplet proximity effect is not immediate in the scattering approach, the method is well suited for the calculation of physical observables, such as the subgap conductance and the Josephson current. Our work thus complements previous studies of the triplet proximity effect in the presence of domain walls in ferromagnets restricted to the limit of weak exchange fields.[14–16]

Although domain walls occur generically in magnetic materials, at first sight they are an unlikely source of the triplet proximity effect in a half-metal: the density of minority carriers decays exponentially away from the superconductor interface so that only domain walls that happen to be adjacent to the interface can contribute to the triplet proximity effect and, of these, only a region of width comparable to the minority-carrier decay length ξ_-. Typically ξ_- is comparable to the (majority) Fermi wavelength λ_F and much smaller than the domain-wall width l_d. This severely restricts the magnitude of the triplet proximity effect mediated by domain walls in the contact geometry of Fig. 1(a) in which a half-metallic film and the superconductor are placed "in series" and the domain wall is parallel to the interface. An additional and not less important complication of the series geometry is that destructive interference between different reflection

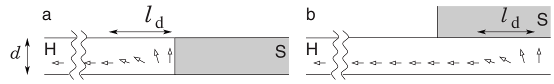

FIG. 1. Superconductor–half-metal junction with a domain wall and serial (a) and lateral (b) contacts.

paths is found to completely suppress the Andreev reflection amplitude at the Fermi level $\varepsilon=0$.[11]

It is the goal of this paper to show that these limitations are absent in a different contact geometry, shown in Fig. 1(b), in which the superconductor is laterally coupled to a magnetic film over a distance much larger than the film thickness d. Although this lateral contact geometry has received as good as no theoretical attention—most theoretical works deal with the serial geometry of Fig. 1(a)—it is the relevant one for the experiment of Ref. 7. We find that for a lateral contact majority electrons have an amplitude r_{he} for Andreev reflection as majority holes that remains finite at the Fermi level and scales proportional to $\lambda_{\text{F}}/\min(l_{\text{d}},d)$. Especially for thin half-metallic films ($d \ll l_{\text{d}}$), the reflection amplitude for a lateral contact is significantly enhanced with respect to the serial geometry for which $r_{\text{he}} \propto \varepsilon\xi_{-}/l_{\text{d}}\Delta$, with Δ being the magnitude of the superconducting order parameter.

In Sec. II below we calculate the Andreev reflection amplitudes. Section III discusses two applications: the two-terminal subgap conductance between the half-metal and the superconductor in the lateral geometry, and the Josephson effect in a superconductor–half-metal–superconductor junction. We conclude in Sec. IV.

II. CALCULATION OF ANDREEV REFLECTION AMPLITUDES

In the lateral contact, the domain wall is perpendicular to the superconductor interface. We first calculate the Andreev reflection amplitude r_{he} in the presence of such a domain wall and then account for the combined effect of multiple Andreev reflections in a thin half-metallic film ($d \lesssim l_{\text{d}}$). Quasiparticle excitations near the interface are described by the Bogoliubov–de Gennes equation

$$\begin{pmatrix} \hat{H} & i\Delta e^{i\phi}\sigma_2 \\ -i\Delta e^{-i\phi}\sigma_2 & -\hat{H}^* \end{pmatrix}\Psi = \varepsilon\Psi, \tag{1}$$

where Ψ is a four-component wave function with components for the electron/hole and spin degrees of freedom and $\Delta e^{i\phi}$ is the superconducting order parameter. We choose coordinates such that the half-metal superconductor interface is the plane $z=0$ and the magnetization direction $\mathbf{m}$ in the half-metal varies in the x direction (see Fig. 2). In the superconductor ($z>0$), we take the Hamiltonian to be

$$\hat{H} = \hat{\mathbf{p}}\frac{1}{2m_{\text{S}}}\hat{\mathbf{p}} - \varepsilon_{\text{F,S}}, \tag{2}$$

where m_{S} and $\varepsilon_{\text{F,S}}=\hbar^2k_{\text{S}}^2/2m_{\text{S}}$ are the effective mass and Fermi energy, respectively. In the half-metal ($z<0$) we set

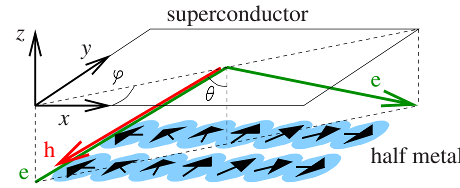

FIG. 2. (Color online) Half-metal superconductor interface with a domain wall. An electron (e) incident on the interface is either normally reflected, or Andreev reflected as a hole (h). The Andreev reflection amplitude r_{he} for this situation given by Eq. (14) of the main text.

$$\hat{H} = \sum_{\pm}\hat{\mathbf{p}}\frac{\hat{P}_{\pm}}{2m_{\pm}}\hat{\mathbf{p}} - \varepsilon_{\text{F},\pm}\hat{P}_{\pm}, \tag{3}$$

where $m_{\pm}$ and $\varepsilon_{\text{F},\pm}=\hbar^2k_{\pm}^2/2m_{\pm}$ are the effective mass and the Fermi energy for majority (+) and minority (−) carriers in the half-metal, and $\hat{P}_{\pm}=(1\pm\mathbf{m}(x)\cdot\boldsymbol{\sigma})/2$ project onto the majority and minority components, respectively. We take the limit $\varepsilon_{\text{F},-}\to-\infty$ so that there are no minority carriers in the half-metal. We further assume that the interface has a normal-state transmission probability $\tau \ll 1$, which we model through the presence of a potential barrier $V\delta(z)$ at the interface.

We choose a right-handed set of unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ and consider a variation in the magnetization direction $\mathbf{m}$ of the form

$$\mathbf{m}(x) = (\mathbf{e}_1\cos\phi_{\text{m}} + \mathbf{e}_2\sin\phi_{\text{m}})\sin\theta_{\text{m}}(x) + \mathbf{e}_3\cos\theta_{\text{m}}(x). \tag{4}$$

We then employ a gauge transformation that rotates $\mathbf{m}$ to the $\mathbf{e}_3$ direction

$$\Psi(x) \to \begin{pmatrix} U^{\dagger}(x) & 0 \\ 0 & U^{\text{T}}(x) \end{pmatrix}\Psi(x), \tag{5}$$

with

$$U(x) = e^{i\theta_{\text{m}}(\mathbf{m}(x)\times\mathbf{e}_3)\cdot\sigma/2\sin\theta_{\text{m}}}. \tag{6}$$

This gauge transformation adds a spin-dependent gauge potential,

$$\mathbf{A} = i\hbar U^{\dagger}\,\boldsymbol{\nabla}\,U, \tag{7}$$

to the Hamiltonian $\hat{H}$,[17] but it does not affect the singlet superconducting order parameter, $U^{\text{T}}i\sigma_2\Delta U=i\sigma_2\Delta$.

Since the domain-wall width l_{d} is typically much larger than the Fermi wavelength, we may neglect spatial variations of $\mathbf{A}$. The wave function Ψ_{e} of an electronic quasiparticle in the half-metal incident on the superconductor then reads as

$$\Psi_{\text{e}}(\mathbf{r}) = \frac{1}{\sqrt{v_{+,z}}}e^{ik_xx+ik_yy}\begin{pmatrix} e^{ik_zz}+r_{\text{ee}}e^{-ik_zz} \\ 0 \\ r_{\text{he}}e^{ik_zz} \\ 0 \end{pmatrix}, \tag{8}$$

where r_{ee} and r_{he} are the amplitudes of normal reflection and Andreev reflection, respectively. Further $k_x=k_+\cos\varphi\sin\theta$,

$k_y = k_+ \sin\varphi \sin\theta$, and $k_z = k_+ \cos\theta = m_+ v_{+,z}/\hbar$, where the polar angles φ and θ parameterize the propagation direction of the electron with respect to the superconductor interface and the domain wall (see Fig. 2). We neglected the small difference of the wave numbers of electrons and holes if the excitation energy ε is finite.

The Andreev reflection amplitude r_{he} can be found by matching Ψ_{e} to a linear combination of the four linearly independent wave functions in the superconductor,

$$\Psi_{\alpha,\beta}(\mathbf{r}) \propto e^{ik_x x + ik_y y + iq(\alpha,\beta)z} \begin{pmatrix} 1 \\ -i\alpha e^{i\phi_{\mathrm{m}}} \\ i\alpha e^{-i\eta(\beta)} \\ e^{-i\eta(\beta) - i\phi_{\mathrm{m}}} \end{pmatrix}, \tag{9}$$

where $\alpha, \beta = \pm 1$, $\eta(\beta) = \phi - \phi_{\mathrm{m}} + \beta \arccos(\varepsilon/\Delta)$, and $q(\alpha,\beta)$ is the solution of

$$q^2 = k_{\mathrm{S}}^2 - k_x^2 - k_y^2 + \frac{2im_{\mathrm{S}}\beta}{\hbar^2}\sqrt{\Delta^2 - \varepsilon^2} - \alpha k_x \frac{\partial\theta_{\mathrm{m}}}{\partial x} \tag{10}$$

with $\mathrm{Im}\, q > 0$. The matching conditions are on the wave function and its derivative at the superconductor interface at $z=0$. The wave function Ψ is continuous,

$$\Psi(z\downarrow 0) = \Psi(z\uparrow 0), \tag{11}$$

whereas its derivative satisfies the equation

$$\frac{\hbar^2}{2m_{\mathrm{S}}} \left.\frac{\partial\Psi}{\partial z}\right|_{z\downarrow 0} = \sum_{\pm} \frac{\hbar^2}{2m_{\pm}} \hat{P}_{\pm} \left.\frac{\partial\Psi}{\partial z}\right|_{z\uparrow 0} + V\Psi(z=0). \tag{12}$$

Since we are interested in the limit $\varepsilon_{\mathrm{F},-} \to -\infty$, for minority components we may replace boundary conditions (11) and (12) with

$$\hat{P}_{-}\Psi_{\mathrm{S}}(z=0) = 0, \tag{13}$$

without a condition on the corresponding derivative. From the resulting six equations we can calculate the six unknowns: two reflection amplitudes and four amplitudes for the wave function in the superconductor.

To lowest order in $\partial\theta_{\mathrm{m}}/\partial x$ and the transmission coefficient τ of the half-metal superconductor interface we then find

$$r_{\mathrm{he}}(\theta,\varphi) = -\frac{\tau(\theta) k_+ \sin\theta \cos\varphi\, e^{-i(\phi-\phi_{\mathrm{m}})}\Delta}{4(k_{\mathrm{S}}^2 - k_+^2 \sin^2\theta)\sqrt{\Delta^2 - \varepsilon^2}} \frac{\partial\theta_{\mathrm{m}}}{\partial x}, \tag{14}$$

where we used the Andreev approximation (which is valid for all angles θ if $k_{\mathrm{S}}^2 \gtrsim k_+^2 \gg \Delta m_{\mathrm{S}}/\hbar^2$) and eliminated the potential barrier V at the interface in favor of the abovementioned transmission coefficient

$$\tau(\theta) = \frac{4\hbar^2 v_{\mathrm{S},z}(\theta) v_{+,z}(\theta)}{4V^2 + \hbar^2[v_{\mathrm{S},z}(\theta) + v_{+,z}(\theta)]^2}, \tag{15}$$

with $m_{\mathrm{S}} v_{\mathrm{S},z} = \hbar(k_{\mathrm{S}}^2 - k_x^2 - k_y^2)^{1/2}$. The amplitude r_{eh} for Andreev reflection of a majority hole into a majority electron is

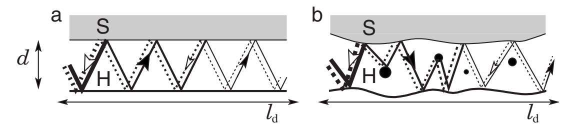

FIG. 3. (a) Ballistic and (b) disordered half-metallic film of thickness d laterally coupled to a superconductor. The Andreev reflection amplitude in the presence of a slowly varying magnetization direction is enhanced by multiple scattering at the superconductor interface.

$$r_{\mathrm{eh}} = r_{\mathrm{he}}^*. \tag{16}$$

The presence of a finite triplet Andreev reflection amplitude at a domain wall is consistent with a previous quasiclassical analysis of the triplet proximity effect at a domain wall in ferromagnets in the limit of weak exchange fields.[15,18] We also note that $r_{\mathrm{he}}^{\mathrm{eff}} \neq 0$ at the Fermi energy is not in contradiction with the observation of Béri *et al.* that $r_{\mathrm{he}} = 0$ at $\varepsilon = 0$ in clean serial half-metal superconductor junctions.[11] For the serial geometry, the Andreev scattering problem may be described using a 2×2 scattering matrix. For the lateral geometry the scattering matrix is intrinsically four dimensional and the argument of Ref. 11 does not apply.

The order of magnitude of reflection amplitude (14) can be understood from the following argument: the amplitude that the incident majority electron is initially reflected into a hole of opposite spin is $\sim\tau(\theta)e^{-i\phi}$. Since the Andreev reflected hole exists up to a distance $\sim 1/k_{\mathrm{S}}$ away from the position of the incident majority carrier,[19] there is a finite overlap with majority hole states in the half-metal. This overlap is proportional to $(\partial\theta_{\mathrm{m}}/\partial x)/k_{\mathrm{S}}$, hence the parameter dependence of Eq. (14).

We now apply the above result to an extended half-metallic film of thickness $d \ll l_{\mathrm{d}}$ laterally coupled to an s-wave superconductor, as in Fig. 1(b). In the thin-film geometry electrons reflect repeatedly off the half-metal superconductor interface [see Fig. 3(a)]. Since the wave functions of the incident electron and the Andreev reflected hole have the same dependence on the position $\mathbf{r}$, see Eq. (8), amplitudes for Andreev scattering from reflections at different positions at the interface add up coherently. This results in an enhancement of the Andreev reflection probability similar in origin to the "reflectionless tunneling effect" in disordered normal-metal superconductor junctions.[20]

We consider a domain wall whose length is shorter than the superconducting coherence length ($l_{\mathrm{d}} \ll \hbar v_+/\Delta$) and for which the orientation of the magnetization varies along the x direction. We assume that the film is in the clean limit (mean free path $\gg l_{\mathrm{d}}$). The scattering states in the film are then parameterized using polar angles θ and φ which set the magnitude of the (now quantized) momentum in the z direction and the propagation direction in the xy plane, respectively. Combining contributions from the entire width of the domain wall, we find that the effective reflection amplitude for Andreev reflection off the domain wall is

$$r_{\rm he}^{\rm eff}(\theta,\varphi) = -\frac{\tau(\theta)k_+ \cos\,\theta e^{-i(\phi-\phi_{\rm m})}\Delta\delta\theta_{\rm m}}{8(k_{\rm S}^2-k_+^2\sin^2\,\theta)d\sqrt{\Delta^2-\varepsilon^2}}\text{sign}(\cos\,\varphi), \tag{17}$$

where $\delta\theta_{\rm m}=\theta_{\rm m}(\infty)-\theta_{\rm m}(-\infty)$ is the total angle by which the magnetization direction changes. The same result is found by directly solving the scattering problem in the thin-film geometry.[21] For thin films, this Andreev reflection amplitude is significantly larger than the single reflection amplitude of Eq. (14). As the final effective amplitude depends only on the total change in angle $\delta\theta_{\rm m}$, $r_{\rm he}^{\rm eff}$ remains finite in the adiabatic limit $l_{\rm d}\to\infty$, despite the vanishing of the rate of change, $\partial\theta_{\rm m}(x)/\partial x \approx \delta\theta_{\rm m}/l_{\rm d}\to 0$.

Equations (14) and (17) are the main results of this paper. As advertised in Sec. I, the Andreev reflection amplitude $r_{\rm he}^{\rm eff}$ is independent of the location of the domain wall, as long as it is "under" the superconducting contact, and the angle of incidence φ. The absence of a dependence on φ implies that the Andreev reflection amplitude does not depend on the orientation of the domain wall. The appearance of the azimuthal angle $\phi_{\rm m}$ in the scattering phase is consistent with the Andreev reflection amplitude found in Ref. 11 for the serial geometry (see also Ref. 22).

III. APPLICATIONS

With the reflection amplitudes obtained above we now consider the conductance $G_{\rm HS}$ of a lateral half-metal superconductor junction [as in Fig. 1(b)] and the Josephson effect in a lateral superconductor–half-metal–superconductor junction [as in Fig. 1(c)]. As before, we consider the case that there is a domain wall somewhere below the superconducting contacts and that the transmission coefficient of the half-metal superconductor interface $\tau \ll 1$. We also assume that the half-metal is in the clean limit,[23] that $k_+ d \gg 1$ (many transverse modes), and that $l_{\rm d} \ll \hbar v_+/\Delta$ (domain wall is short in comparison to the superconducting coherence length). In order to simplify our final expressions, we set $k_{\rm S}=k_+$. For the subgap conductance $G_{\rm HS}(V)=\partial I/\partial V$ we then find

$$\begin{aligned} G_{\rm HS}(V) &= \frac{2e^2}{h}\text{tr}\, r_{\rm he}^{\rm eff}(eV) r_{\rm he}^{\rm eff}(eV)^\dagger \\ &= \frac{e^2 W}{hd}\frac{\langle \tau(\theta)^2\rangle \Delta^2}{64\pi(\Delta^2-e^2V^2)}(\delta\theta_{\rm m})^2, \end{aligned} \tag{18}$$

where W is the width of the half-metallic film and the brackets $\langle \ldots \rangle$ denote an angular average. This result is to be contrasted with the conductance of a half-metal superconductor junction with a domain wall parallel to the interface in the serial geometry, which is proportional to

$$G_{\rm HS}(V) \propto \frac{e^2 W d}{h l_{\rm d}^2}\frac{e^2V^2}{\Delta^2}(\delta\theta_{\rm m})^2 \tag{19}$$

if $eV \ll \Delta$.[11]

When calculating the Josephson effect, we take the junction to be reflection symmetric, with a domain wall below each superconductor such that the azimuthal angles $\phi_{\rm m}$ and the angle changes $\delta\theta_{\rm m}$ are equal (see Fig. 4). We then calculate the zero-temperature supercurrent from the expression[24]

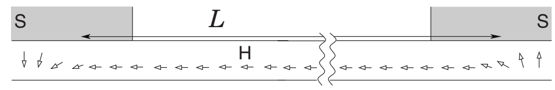

FIG. 4. Superconductor–half-metal–superconductor junction with a domain walls and lateral contacts.

$$I = -\frac{2e}{\pi\hbar}\frac{\partial}{\partial\phi}\text{Re}\int_0^\infty d\omega \text{tr}\,\ln[1+e^{-2\omega L/\hbar v}|r_{\rm he}^{\rm eff}(i\omega)|^2 e^{i\phi}], \tag{20}$$

where v is the propagation velocity of a transverse mode, L is the distance between the domain walls, and ϕ is the phase difference between the superconducting order parameters. For short junctions, $L \ll \hbar v_+/\Delta$, we then find

$$eI = \pi G_{\rm HS}(0)\Delta\,\sin\,\phi, \tag{21}$$

where $G_{\rm HS}(0)$ is the Fermi-level conductance of a single half-metal superconductor interface given in Eq. (18) above. For a long junction, $L \gg \hbar v_+/\Delta$ one has

$$eI = \frac{8}{15}G_{\rm HS}(0)\frac{\hbar v_+}{L}\sin\,\phi, \tag{22}$$

where $v_+=\hbar k_+/m_+$. We note that the long-junction limit of supercurrent (22) is parametrically larger than the supercurrent in a serial geometry, which scales proportional to[11]

$$eI \propto \frac{\hbar^3 v_+^3}{L^3\Delta^2}. \tag{23}$$

The junction becomes a "π junction," with a supercurrent proportional to $-\sin\,\phi$, if the two domain walls have opposite $\delta\theta_{\rm m}$.[25]

IV. CONCLUSION

Although the calculations presented in this paper are for ballistic half-metal superconductor junctions, we expect that the enhanced tripled proximity effect in the lateral geometry also exists in the presence of disorder in the same way as reflectionless tunneling exists both in clean and disordered junctions.[20] As long as the non-Andreev reflected electron is transmitted through the domain wall, as in Fig. 3(b), the coherent addition of amplitudes from multiple Andreev reflections is not affected by changes of the electron's propagation direction in a disordered half-metallic film. We have thus identified a mechanism by which domain walls in a lateral geometry contribute to the long-range proximity effect irrespective of their position (as long as they are under the superconducting contact), their orientation, and their width.

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